

CONSTRUCTION OF FUZZY-CORRECT MODEL OF DECISION-MAKING TASKS

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ABSTRACT

Systematically analyzed the task of constructing of models of fuzzy assessment tasks and forecasting of weakly formalized processes. It is shown that in the process of building such models appear incorrect tasks. Investigated compact and noncompact classes of accuracy. Showed possibility of getting fuzzy and unclear sustainable solutions of incorrect tasks during constructing models for risk assessment using a variety of membership functions.

KEYWORDS: Fuzzy Set, Risk Assessment, Risk Forecasting, Decision-Making, Membership Function, Incorrect Task

1. INTRODUCTION

The problem of constructing fuzzy models based on the findings of fuzzy rules in the conditions of uncertainty arises in assessing the state of weakly formalized processes. The advantage of fuzzy logic consists in the possibility of the use of expert knowledge about the object in the form of statements (predicate rules): if - «inputs», then «outputs». However, it should be noted that the building of fuzzy models of this type often associated with the advent of so-called fuzzy-incorrect tasks.

It should be particularly noted the work of A. N. Tikhonova, V. Ya. Arsenin and A. V. Yazenin in finding approximate solutions of incorrect tasks at constructing formalized assessment models. However, the questions of solutions of fuzzy – incorrect tasks currently insufficiently studied.

2. STATEMENT OF THE TASK

Let the state of weakly formalized process is described by sampling fuzzy experimental data (X_r, y_r) , $r = \overline{1, M}$. Here $X_r = (x_{r1}, x_{r2}, \dots, x_{rm})$ - input n - dimensional fuzzy vector, that is set with its functions facilities, and $y_r = (y_1, y_2, \dots, y_m)$ - corresponding to it output vector.

Should build a fuzzy-correct model of decision making by assessing and forecasting the state of weakly formalized process described set of fuzzy rules productions (linguistic utterances) similar to the following

$$\bigcup_{p=1}^{k_i} \left(\bigcap_{j=1}^n x_j^i = a_{ij}^p - \text{with the weight } w_{ip} \right) \rightarrow y_i^f = f(b_{i0}, b_{i1}, b_{i2}, \dots, b_{in}), \quad i = \overline{1, m}. \quad (1)$$

Here: a_{ij}^p - linguistic term, which is estimated variable x_j^i in line with the number p in the rules i ;

w_{ip} - weight coefficient for lines p in the rules i ;

$y_i^f = f(b_{i0}, b_{i1}, b_{i2}, \dots, b_{in})$ - the output of the model (1), as described in the rule i .

Needed to find the values of unknown coefficients $b_{ij} (i = \overline{1, m}, j = \overline{1, n})$ in the process of building a fuzzy model (1), which provide the minimum residual:

$$E = \sum_{r=1}^M (y_r - y_r^f) \rightarrow \min, \quad (2)$$

where y_r^f - the output of the model (1), corresponding to input vector X_r .

3. CONSTRUCTION FUZZY MODEL OF DECISION TASKS BY ESTIMATION AND FORECASTING OF THE STATE OF WEAKLY FORMALIZED PROCESS

Let's consider three types of studied fuzzy models (1) - outputs which can be represented as linear and nonlinear dependencies, as well as in the form of fuzzy terms.

- Fuzzy model, the output of which is represented as a linear dependence, is giving by Sugeno type fuzzy sets model:

If $(x_1^i = a_{i1}^1 \wedge x_2^i = a_{i2}^1 \wedge \dots \wedge x_n^i = a_{in}^1)$ (with the weight w_{i1}) \vee

$\vee (x_1^i = a_{i1}^{k_i} \wedge x_2^i = a_{i2}^{k_i} \wedge \dots \wedge x_n^i = a_{in}^{k_i})$ (with the weight w_{ik_i}),

$$\text{then } y_i^f = b_{i0} + b_{i1} \frac{\sum_{j=1}^q \mu(x_1^{ij}) x_1^{ij}}{\sum_{j=1}^q \mu(x_1^{ij})} + b_{i2} \frac{\sum_{j=1}^q \mu(x_2^{ij}) x_2^{ij}}{\sum_{j=1}^q \mu(x_2^{ij})} + \dots + b_{in} \frac{\sum_{j=1}^q \mu(x_n^{ij}) x_n^{ij}}{\sum_{j=1}^q \mu(x_n^{ij})}, \quad i = \overline{1, m}.$$

Here q - the cardinality of the input variables x_j^i in this model.

- Fuzzy model, the output of which is represented in the form of the nonlinear dependence, specified as:

If $(x_1^i = a_{i1}^1 \wedge x_2^i = a_{i2}^1 \wedge \dots \wedge x_n^i = a_{in}^1)$ (with the weight w_{i1}) \vee

$\vee (x_1^i = a_{i1}^{k_i} \wedge x_2^i = a_{i2}^{k_i} \wedge \dots \wedge x_n^i = a_{in}^{k_i})$ (with the weight w_{ik_i}),

then

$$y_i^f = b_{i0} + b_{i1} \frac{\sum_{j=1}^q \mu(x_1^{ij}) x_1^{ij}}{\sum_{j=1}^q \mu(x_1^{ij})} + b_{i2} \frac{\sum_{j=1}^q \mu(x_2^{ij}) x_2^{ij}}{\sum_{j=1}^q \mu(x_2^{ij})} + \dots + b_{in} \frac{\sum_{j=1}^q \mu(x_n^{ij}) x_n^{ij}}{\sum_{j=1}^q \mu(x_n^{ij})} +$$

$$+ b_{in+1} \left[\frac{\sum_{j=1}^q \mu(x_1^{ij}) x_1^{ij}}{\sum_{j=1}^q \mu(x_1^{ij})} \right]^2 + b_{in+2} \left[\frac{\sum_{j=1}^q \mu(x_2^{ij}) x_2^{ij}}{\sum_{j=1}^q \mu(x_2^{ij})} \right]^2 + \dots + b_{i2n} \left[\frac{\sum_{j=1}^q \mu(x_n^{ij}) x_n^{ij}}{\sum_{j=1}^q \mu(x_n^{ij})} \right]^2, \quad i = \overline{1, m}.$$

- Fuzzy model, the output of which is represented in the form of fuzzy terms, set the fuzzy Mamadani type model:

If $(x_1^i = a_{i1}^1 \wedge x_2^i = a_{i2}^1 \wedge \dots \wedge x_n^i = a_{in}^1)$ (with the weight w_{i1}) \vee

$\vee (x_1^i = a_{i1}^{k_i} \wedge x_2^i = a_{i2}^{k_i} \wedge \dots \wedge x_n^i = a_{in}^{k_i})$ (with the weight w_{ik_i}),

then $y_i^f = term_i$, $i = \overline{1, m}$.

In these three types of models to X_r input vector corresponds to the next result of fuzzy output:

$$y_r^f = \frac{\sum_{i=1}^m \mu_{d_i}(X_r) \cdot d_i}{\sum_{i=1}^m \mu_{d_i}(X_r)} \quad \text{or} \quad y_r^f = \frac{\int_{\underline{z}}^{\overline{z}} \mu_z(X_r) z dz}{\int_{\underline{z}}^{\overline{z}} \mu_z(X_r) dz} \quad (3)$$

where

$$d_i = \begin{cases} b_{i0} + b_{i1}x_1^r + b_{i2}x_2^r + \dots + b_{in}x_n^r, & \text{in the case of linear dependence,} \\ b_{i0} + b_{i1}x_1^r + b_{i2}x_2^r + \dots + b_{in}x_n^r + b_{in+1}(x_1^r)^2 + b_{in+2}(x_2^r)^2 + \dots + b_{i2n}(x_n^r)^2, & \text{in the case of nonlinear} \\ term_i, & \text{dependence,} \\ & \text{case of fuzzy terms;} \end{cases}$$

Conclusion of the i -rule in discrete case;

$$z = \begin{cases} b_0 + b_1x_1^r + b_2x_2^r + \dots + b_nx_n^r, & \text{in the case of linear dependence,} \\ b_0 + b_1x_1^r + b_2x_2^r + \dots + b_nx_n^r + b_{n+1}(x_1^r)^2 + b_{n+2}(x_2^r)^2 + \dots + b_{2n}(x_n^r)^2, & \text{in the case of nonlinear} \\ term, & \text{dependence,} \\ & \text{in the case of fuzzy terms;} \end{cases}$$

Conclusion of i -rule in the smooth case;

$\mu_{d_i}(X_r)$ - membership function, corresponding to the values of the input vector X_r of conclusion d_i in the discrete case,

$\mu_z(X_r)$ - membership function, corresponding to the values of the input vector X_r of conclusion z smooth case.

We introduce the designation

$$\beta_{ir} = \frac{\mu_{d_i}(X_r) \cdot d_i}{\sum_{i=1}^m \mu_{d_i}(X_r)}, \quad \gamma_r = \frac{\mu_z(X_r) \cdot z}{\int_{\underline{z}}^{\overline{z}} \mu_z(X_r) dz},$$

with regard to which the expression (3) rewritten in the form:

$$y_r^f = \sum_{i=1}^m \beta_{ir} d_i = \begin{cases} \sum_{i=1}^m (\beta_{ir} \cdot b_{i0} + \beta_{ir} \cdot b_{i1} \cdot x_1^r + \dots + \beta_{ir} \cdot b_{in} \cdot x_n^r), & \text{in the case of linear dependence} \\ \sum_{i=1}^m (\beta_{ir} \cdot b_{i0} + \dots + \beta_{ir} \cdot b_{in} \cdot x_n^r + \beta_{ir} \cdot b_{in+1} \cdot (x_1^r)^2 + \dots + \beta_{ir} \cdot b_{i2n} \cdot (x_n^r)^2), & \text{in the case of nonlinear dependence} \\ \sum_{i=1}^m (\beta_{ir} \cdot \text{term}), & \text{in the case of fuzzy terms} \end{cases}$$

In discrete case or

$$y_r^f = \int_{\underline{z}}^{\bar{z}} \gamma_r dz$$

In smooth case

Let's introduce the following designation:

$$Y^f = (y_1^f, y_2^f, \dots, y_M^f)^T;$$

$$Y = (y_1, y_2, \dots, y_M)^T;$$

$$A = \begin{bmatrix} \beta_{1,1}, \dots, \beta_{1,m}, & x_{1,1} \cdot \beta_{1,1}, \dots, x_{1,1} \cdot \beta_{1,m}, & \dots, & x_{1,n} \cdot \beta_{1,1}, \dots, x_{1,n} \cdot \beta_{1,m} \\ \vdots & & & \\ \beta_{M,1}, \dots, \beta_{M,m}, & x_{M,1} \cdot \beta_{1,1}, \dots, x_{M,1} \cdot \beta_{1,m}, & \dots, & x_{M,n} \cdot \beta_{M,1}, \dots, x_{M,n} \cdot \beta_{M,m} \end{bmatrix}.$$

Taking into account these designations expression (2) rewrite the following matrix form:

$$E = (Y - Y^f)^T \cdot (Y - Y^f) \rightarrow \min. \quad (4)$$

$$\text{where } Y^f = A \cdot B.$$

The minimum value of squared residuals E is achieved at $Y^f = Y$, that corresponds to the solution of the equation

$$Y = A \cdot B. \quad (5)$$

The construction of the sought fuzzy model is reduced to a finding of such a vector, which is the condition (4). In proposed models each input value has its own function facilities $\mu(x, c, \sigma)$ with fuzzy terms (for example, L - low, PI- pre-intermediate, I-intermediate, UI – upper intermediate, A-advanced) used in equations.

During the development of the fuzzy model-based on risk assessment conclusions of fuzzy rules are often faced with the problem of finding of approximate solutions of fuzzy incorrect tasks. It should be noted that the methods for solving of incorrect tasks of decision support systems, designed with just a series of individual cases models (for example, for models based on classical logic). However, a common approach to fuzzy – incorrect tasks of this type do

not currently exist. To solve this task, you can use the method of finding the vicinity of the solution of incorrect tasks. With this purpose we need some definitions and prove approval.

Definition 1: Primary information (about the study) is a fuzzy set with the function of toiletries $\mu(x)$, where $x \in X$.

Definition 2: Primary information is called a fuzzy-compact, if any level set, except the null will be compact in the space of X , i.e.

$$\forall \alpha \in (0,1], A_\alpha = \{x: \mu(x) \geq \alpha\} \text{—compact domain in the space.}$$

Approval 1: Primary information, the membership function of which is shown in the table. 1, is unclear-compact primary information.

Table 1

№	Membership Function	Proof of Fuzzy Compactness of Primary Information
1.	$\mu(x) = e^{-k\ x\ }$	$\forall \alpha \in (0,1], k > 1, 0 \leq x < \infty \quad A_\alpha(x) = \{x: \mu(x) \geq \alpha\} \Rightarrow$ $A_\alpha(x) = \{x: e^{-k\ x\ } \geq \alpha\} = \{x: -k\ x\ \geq \ln \alpha\} =$ $\left\{x: \ x\ \leq -\frac{\ln \alpha}{k}\right\} = \{x: \ x\ < \varepsilon(\alpha)\},$ $\varepsilon(\alpha) = -\frac{\ln \alpha}{k}$
2.	$\mu(x) = e^{-k\ x\ ^2}$	$\forall \alpha \in (0,1], k > 0, \quad A_\alpha(x) = \{x: \mu(x) \geq \alpha\} \Rightarrow$ $A_\alpha(x) = \{x: e^{-k\ x\ ^2} \geq \alpha\} = \{x: -k\ x\ ^2 \geq \ln \alpha\} =$ $\left\{x: \ x\ ^2 \geq -\frac{\ln \alpha}{k}\right\} = \left\{x: \ x\ \leq \sqrt{-\frac{\ln \alpha}{k}}\right\} =$ $\{x: \ x\ < \varepsilon(\alpha)\}, \quad \varepsilon(\alpha) = \sqrt{-\frac{\ln \alpha}{k}}$
3.	$\mu(x) = \frac{1}{1+k\ x\ ^2}$	$\forall \alpha \in (0,1], k > 1, \quad A_\alpha(x) = \{x: \mu(x) \geq \alpha\} \Rightarrow$ $A_\alpha(x) = \left\{x: \frac{1}{1+k\ x\ ^2} \geq \alpha\right\} = \left\{x: 1+k\ x\ ^2 \leq \frac{1}{\alpha}\right\} =$ $\left\{x: \ x\ ^2 \leq \frac{1-\alpha}{k\alpha}\right\} = \left\{x: \ x\ \leq \sqrt{\frac{1-\alpha}{k\alpha}}\right\} =$ $\{x: \ x\ < \varepsilon(\alpha)\}, \quad \varepsilon(\alpha) = \sqrt{\frac{1-\alpha}{k\alpha}}$

Here

$$\|x\| = \sqrt{\sum_{i=1}^n x_i^2}.$$

The statement is proved.

Search of solutions of the equations $AB = Y$ is reduced to the problem of search of the fuzzy solution of this equation.

Definition 3. Fuzzy solution to the equation is called the primary information provided by fuzzy set $\bigcup_{\alpha} \alpha A_{\alpha}$, with the following properties:

- given operator A and the original data B ;
- $\forall \alpha \in (0,1], A_{\alpha} = \{B : \mu_A(B) \geq \alpha\}$;

$$\exists \varepsilon(\alpha) > 0, \sup_{B \in A_{\alpha}} \rho_B(A(B), A_{\alpha}) < \varepsilon(\alpha) < \infty.$$

Here ρ_B – interval between sets $A(B)$ and A_{α} .

Definition 4. Fuzzy decision will be called sustainable if $\lim_{\alpha \rightarrow 1} \sup_{x \in X} \mu(x) \varepsilon(\alpha) = 0$ and operator $A : B \rightarrow Y$

continuous in $B \in Z$,

Approval 2. Fuzzy solution of equations (4) with the facilities of input variables listed in the table. 2, is sustainable:

Table 2

№	Membership Function	Proof of Fuzzy Compactness of Primary Information
1.	$\mu(B) = e^{-k\ AB-Y\ }$	$\forall \alpha \in (0,1], k > 1,$ $\varepsilon(\alpha) = -\frac{\ln \alpha}{k},$ $\lim_{\alpha \rightarrow 1} \varepsilon(\alpha) = \lim_{\alpha \rightarrow 1} \left(-\frac{\ln \alpha}{k} \right) = 0$
2.	$\mu(B) = e^{-k\ AB-Y\ ^2}$	$\forall \alpha \in (0,1], k > 0,$ $\varepsilon(\alpha) = \sqrt{-\frac{\ln \alpha}{k}},$ $\lim_{\alpha \rightarrow 1} \varepsilon(\alpha) = \lim_{\alpha \rightarrow 1} \left(\sqrt{-\frac{\ln \alpha}{k}} \right) = 0$
3.	$\mu(B) = \frac{1}{1+k\ AB-Y\ ^2}$	$\forall \alpha \in (0,1], k > 1,$ $\varepsilon(\alpha) = \sqrt{\frac{1-\alpha}{k\alpha}},$ $\lim_{\alpha \rightarrow 1} \varepsilon(\alpha) = \lim_{\alpha \rightarrow 1} \left(\sqrt{\frac{1-\alpha}{k\alpha}} \right) = 0$

4.	$\mu(B) = \begin{cases} 0, & -\infty < (AB-Y) \leq -\frac{1}{\sqrt[k]{a}}, \\ 1-a(-(AB-Y)^k), & -\frac{1}{\sqrt[k]{a}} \leq (AB-Y) \leq 0, \\ 1-a(AB-Y)^k, & 0 \leq (AB-Y) \leq \frac{1}{\sqrt[k]{a}}, \\ 0, & \frac{1}{\sqrt[k]{a}} \leq (AB-Y) < \infty. \end{cases}$	$\forall \alpha \in (0,1], -\frac{1}{\sqrt[k]{a}} \leq (AB-Y) \leq \frac{1}{\sqrt[k]{a}},$ $\varepsilon(\alpha) = \sqrt[k]{\frac{1-\alpha}{a}},$ $\lim_{\alpha \rightarrow 1} \varepsilon(\alpha) = \lim_{\alpha \rightarrow 1} \left(\sqrt[k]{\frac{1-\alpha}{a}} \right) = 0$
5.	$\mu(z) = \begin{cases} 0, & -\infty < (AB-Y) \leq -a_2, \\ \frac{a_2 + (AB-Y)}{a_2 - a_1}, & -a_2 \leq (AB-Y) \leq -a_1, \\ 1, & -a_1 \leq (AB-Y) \leq a_1 \\ \frac{a_2 - (AB-Y)}{a_2 - a_1}, & a_1 \leq (AB-Y) \leq a_2, \\ 0, & a_2 \leq (AB-Y) < \infty. \end{cases}$	$\forall \alpha \in (0,1], -a_2 \leq (AB-Y) \leq a_2,$ $\varepsilon(\alpha) = a_2 - (a_2 - a_1)\alpha,$ <p>Equal to</p> $\lim_{\alpha \rightarrow 1} \varepsilon(\alpha) = \lim_{\alpha \rightarrow 1} (a_2 - (a_2 - a_1)\alpha) = 0$ <p>only at $a_1 \rightarrow 0$</p>

Often a criterion characterizing the degree of precision of the solution of tasks and allocation of class correctness of the incorrect task is not enough for good fuzzy solutions. To build a more adequate model is more convenient to consider additional knowledge about the decision as some of the *primary information*.

Let the task is to find the solutions of integral Fredholm equations of the first kind

$$Az = \int_a^b K(x, s)z(s)ds = u_\delta(x).$$

$$\text{Space } Z = L_2[a, b], U = C[c, d].$$

Construct a fuzzy solution in a standard way. As the membership function of the concept of «the accuracy of the model» take

$$\mu_A(z) = \exp(-\|Az - u_\delta\|).$$

As additional information, take a fuzzy set, which corresponds to the concept of «small by norm in $W_2^1[0,1]$ function», i.e. small function and its derivative $\mu_B(z) = \exp(-\|z\|_{W_2^1}^2)$,

$$\text{where } \|z\|_{W_2^1} = \int_0^1 \left[z^2(s) + \left(\frac{\partial z(s)}{\partial s} \right)^2 \right] ds.$$

It is easy to prove that this primary information fuzzy-compact.

Let's construct a fuzzy solution to this task.

$$\mu(z) = \mu_A^\eta(z) \mu_B(z).$$

To obtain an ordinary resolution of the applicable information operator

$$\begin{aligned} I\mu(z) &= \left\{ z : z = \arg \sup_{z \in Z} \mu(z) \right\} = \\ &= \left\{ z : z = \arg \inf_{z \in Z} \left[\eta \int_0^1 K(x, s) z(s) ds - u_\delta + \int_0^1 \left(z^2(s) + \left(\frac{\partial z(s)}{\partial s} \right)^2 \right) ds \right] \right\}. \end{aligned} \quad (6)$$

Here, believing that

$$\int_c^d \left[\int_a^b K_h(x, s) z(s) ds - u_\delta \right]^2 dx \approx \sum_{i=1}^m \left[\sum_{j=1}^n a_{ij} z_j h_s - u_i \right]^2 h_x,$$

$$a_{ij} = K_h(x_i, s_j) \quad j = 2, \dots, n-1, \quad a_{ij} = K_h(x_i, s_j) / 2 \quad j = 1, \text{ Ba } j=n,$$

$$\int_a^b [z(s)]^2 ds \approx \sum_{j=1}^n z_j^2 h_s, \quad \int_a^b [z'(s)]^2 ds \approx \sum_{j=1}^n \frac{(z_j - z_{j-1})^2}{h_s^2} \cdot h_s = \sum_{j=1}^n \frac{(z_j - z_{j-1})^2}{h_s},$$

Write (6) in the form of

$$\begin{aligned} I\mu(z) &= \left\{ z : z = \arg \inf_{z \in Z} \left[\eta \int_0^1 K(x, s) z(s) ds - u_\delta + \int_0^1 \left(z^2(s) + \left(\frac{\partial z(s)}{\partial s} \right)^2 \right) ds \right] \right\} = \\ &= \left\{ z : z = \arg \inf_{z \in Z} \left[\sum_{i=1}^n \left[\sum_{j=1}^n a_{ij} z_j h_s - u_i \right]^2 h_x + \beta \sum_{j=1}^n z_j^2 h_s + \beta \sum_{j=1}^n (z_j - z_{j-1})^2 / h_s \right] \right\} = \\ &= \left\{ z : z = \arg \inf_{z \in Z} \left[[a_{11} z_1 h_s + a_{12} z_2 h_s + \dots + a_{1k} z_k h_s + \dots + a_{1n} z_n h_s - u_1]^2 h_x + \right. \right. \\ &\quad + [a_{21} z_1 h_s + a_{22} z_2 h_s + \dots + a_{2k} z_k h_s + \dots + a_{2n} z_n h_s - u_2]^2 h_x + \\ &\quad + \dots + \\ &\quad + [a_{k1} z_1 h_s + a_{k2} z_2 h_s + \dots + a_{kk} z_k h_s + \dots + a_{kn} z_n h_s - u_k]^2 h_x + \\ &\quad + \dots + \\ &\quad + [a_{n1} z_1 h_s + a_{n2} z_2 h_s + \dots + a_{nk} z_k h_s + \dots + a_{nn} z_n h_s - u_n]^2 h_x + \beta [z_1^2 h_s + z_2^2 h_s + \dots + z_k^2 h_s] + \\ &\quad \left. + \beta \left[\frac{(z_2 - z_1)^2}{h_s} + \frac{(z_3 - z_2)^2}{h_s} + \dots + \frac{(z_{k+1} - z_k)^2}{h_s} + \dots + \frac{(z_{n+1} - z_n)^2}{h_s} \right] \right] \right\} = \end{aligned}$$

$$\left\{ z : z = \arg \left[a_{11} z_1 h_s + a_{12} z_2 h_s + \dots + a_{1k} z_k h_s + \dots + a_{1n} z_n h_s - u_1 \right] a_{1k} h_x h_s + \right. \\
+ 2 \left[a_{21} z_1 h_s + a_{22} z_2 h_s + \dots + a_{2k} z_k h_s + \dots + a_{2n} z_n h_s - u_2 \right] a_{2k} h_x h_s + \\
+ \dots + \\
+ \left[a_{k1} z_1 h_s + a_{k2} z_2 h_s + \dots + a_{kk} z_k h_s + \dots + a_{kn} z_n h_s - u_k \right]^2 a_{kk} h_x h_s + \\
+ \dots + \\
+ \left[a_{n1} z_1 h_s + a_{n2} z_2 h_s + \dots + a_{nk} z_k h_s + \dots + a_{nn} z_n h_s - u_n \right]^2 h_x + \\
\left. + 2\alpha z_k h_s - \frac{2}{h_s} \alpha (z_{k+1} - z_k) = 0 \right\}.$$

From there

$$I\mu(z) = \left\{ z : z = \arg \left[h_x h_s \sum_{k=1}^n \left[\sum_{i=1}^m a_{ik} a_{i1} \right] z_1 + \beta h_s z_1 - \beta \frac{z_2 - z_1}{h_s} = \sum_{i=1}^m a_{i1} u_i h_x h_s, \right. \right. \\
h_x h_s \sum_{k=1}^n \left[\sum_{i=1}^m a_{ik} a_{ij} \right] z_k + \beta h_s z_j - \beta \frac{z_{j+1} - 2z_j + z_{j-1}}{h_s} = \sum_{i=1}^m a_{ij} u_i h_x h_s, \quad j = 2, \dots, n-1, \\
h_x h_s \sum_{k=1}^n \left[\sum_{i=1}^m a_{ik} a_{in} \right] z_k + \beta h_s z_n - \beta \frac{z_{n+1} - 2z_n}{h_s} = \sum_{i=1}^m a_{in} u_i h_x h_s \left. \right\} \Rightarrow \\
I\mu(z) = \left\{ z : z = \arg \left[\sum_{i=1}^m b_{jk} z_k + \sum_{k=1}^n c_{ik} z_k \right] = d_i \right\}.$$

Here:

$$h_x \left[\sum_{i=1}^m a_{ik} a_{ij} \right] = b_{jk}, \quad z_j \left[\beta + \frac{2\beta}{h_s^2} \right] - \frac{2\beta}{h_s^2} z_{j+1} - \frac{\beta}{h_s^2} z_1.$$

Approval 3. Let's operator $A : B \rightarrow Y$ continuous in $B \in Z$, then you can build a stable, fuzzy solution with a function of the facilities specified in the Table. 3.

Table 3

№	Membership Function	Proof of Fuzzy Compactness of Primary Information
1.	$\mu(B) = e^{-k\ AB-Y\ }$	$\forall \alpha \in (0,1], k > 1, 0 \leq B < \infty,$ $\varepsilon(\alpha) = -\frac{\ln \alpha}{k},$ You can build a fuzzy-sustainable solution in the form $A_\alpha = O_{\varepsilon(\alpha)}(AB), \bigcup_{\alpha} \alpha A_\alpha, \lim_{\alpha \rightarrow 1} \varepsilon(\alpha) = 0$

2.	$\mu(B) = e^{-k\ AB-Y\ ^2}$	$\forall \alpha \in (0,1], k > 1,$ $\varepsilon(\alpha) = \sqrt{-\frac{\ln \alpha}{k}},$ You can build a fuzzy-sustainable solution in the form $A_\alpha = O_{\varepsilon(\alpha)}(AB), \bigcup_{\alpha} \alpha A_\alpha, \lim_{\alpha \rightarrow 1} \varepsilon(\alpha) = 0$
3.	$\mu(B) = \frac{1}{1+k\ AB-Y\ ^2}$	$\forall \alpha \in (0,1], k > 1,$ $\varepsilon(\alpha) = \sqrt{\frac{1-\alpha}{k\alpha}},$ You can build a fuzzy-sustainable solution in the form $A_\alpha = O_{\varepsilon(\alpha)}(AB), \bigcup_{\alpha} \alpha A_\alpha, \lim_{\alpha \rightarrow 1} \varepsilon(\alpha) = 0$
4.	$\mu(z) = \begin{cases} 0, & -\infty < (AB-Y) \leq -\frac{1}{\sqrt[k]{a}}, \\ 1-a(-(AB-Y)^k), & -\frac{1}{\sqrt[k]{a}} \leq (AB-Y) \leq 0, \\ 1-a(AB-Y)^k, & 0 \leq (AB-Y) \leq \frac{1}{\sqrt[k]{a}}, \\ 0, & \frac{1}{\sqrt[k]{a}} \leq (AB-Y) < \infty. \end{cases}$	$\forall \alpha \in (0,1], -\frac{1}{\sqrt[k]{a}} \leq B \leq 0, 0 \leq B \leq \frac{1}{\sqrt[k]{a}},$ $\varepsilon(\alpha) = \sqrt[k]{\frac{1-\alpha}{a}},$ You can build a fuzzy-sustainable solution in the form $A_\alpha = O_{\varepsilon(\alpha)}(AB), \bigcup_{\alpha} \alpha A_\alpha, \lim_{\alpha \rightarrow 1} \varepsilon(\alpha) = 0$
5.	$\mu(z) = \begin{cases} 0, & -\infty < (AB-Y) \leq -a_2, \\ \frac{a_2+(AB-Y)}{a_2-a_1}, & -a_2 \leq (AB-Y) \leq -a_1, \\ 1, & -a_1 \leq (AB-Y) \leq a_1 \\ \frac{a_2-(AB-Y)}{a_2-a_1}, & a_1 \leq (AB-Y) \leq a_2, \\ 0, & a_2 \leq (AB-Y) < \infty. \end{cases}$	$\forall \alpha \in (0,1], -a_2 \leq B < -a_1, a_1 \leq B < a_2$ $\varepsilon(\alpha) = a_2 - (a_2 - a_1)\alpha,$ You can build a fuzzy-sustainable solution in the form $A_\alpha = O_{\varepsilon(\alpha)}(AB), \bigcup_{\alpha} \alpha A_\alpha, \lim_{\alpha \rightarrow 1} \varepsilon(\alpha) = 0$

The above mechanism of construction of the fuzzy-specific models can be used for solve the tasks of parametrical identification, classification, clustering and forecasting [3].

4. NUMERICAL EXPERIMENT

The approach has been tested by solving the task of evaluation and prediction using real data (for example, the risk of shortfalls in crop).

The task of computing experiment consists from implementation and analysis of the correctness of the proposed algorithm for solving the decision on assessment of the status of weakly formalized processes described by fuzzy - neural network. The input data of the model were presented by unmanaged parameters: x_1 - weather conditions during sowing, x_2 - water supply, x_3 - weather conditions during the growing season, x_4 - weather conditions during harvesting.

Developed three types of models to assess the risk of shortfalls in crop based on fuzzy rules of the output.

- The risk assessment model, the output of which is expressed by linear dependence.

If $x_1^1=H$ and $x_2^1=H$ and $x_3^1=H$ and $x_4^1=C$

$$\text{Then } y_1^f = 0,33 - 0,05 \frac{\sum_{j=1}^n \mu(x_1^{1j})x_1^{1j}}{\sum_{j=1}^n \mu(x_1^{1j})} - 0,02 \frac{\sum_{j=1}^n \mu(x_2^{1j})x_2^{1j}}{\sum_{j=1}^n \mu(x_2^{1j})} - 0,21 \frac{\sum_{j=1}^n \mu(x_3^{1j})x_3^{1j}}{\sum_{j=1}^n \mu(x_3^{1j})} - 0,1 \frac{\sum_{j=1}^n \mu(x_4^{1j})x_4^{1j}}{\sum_{j=1}^n \mu(x_4^{1j})}.$$

If $x_1^2=H$ and $x_2^2=H$ and $x_3^2=C$ and $x_4^2=C$

$$\text{then } y_2^f = 0,257 - 0,0393 \frac{\sum_{j=1}^n \mu(x_1^{2j})x_1^{2j}}{\sum_{j=1}^n \mu(x_1^{2j})} - 0,112 \frac{\sum_{j=1}^n \mu(x_4^{2j})x_4^{2j}}{\sum_{j=1}^n \mu(x_4^{2j})}.$$

If $x_1^3=H$ and $x_2^3=C$ and $x_3^3=H$ and $x_4^3=C$

$$\text{Then } y_3^f = 0,18 - 0,01 \frac{\sum_{j=1}^n \mu(x_1^{3j})x_1^{3j}}{\sum_{j=1}^n \mu(x_1^{3j})} - 0,07 \frac{\sum_{j=1}^n \mu(x_2^{3j})x_2^{3j}}{\sum_{j=1}^n \mu(x_2^{3j})} - 0,05 \frac{\sum_{j=1}^n \mu(x_3^{3j})x_3^{3j}}{\sum_{j=1}^n \mu(x_3^{3j})} - 0,111 \frac{\sum_{j=1}^n \mu(x_4^{3j})x_4^{3j}}{\sum_{j=1}^n \mu(x_4^{3j})}.$$

If $x_1^4=H$ and $x_2^4=C$ and $x_3^4=C$ and $x_4^4=C$

$$\text{then } y_4^f = 0,26 - 0,02 \frac{\sum_{j=1}^n \mu(x_1^{4j})x_1^{4j}}{\sum_{j=1}^n \mu(x_1^{4j})} - 0,05 \frac{\sum_{j=1}^n \mu(x_2^{4j})x_2^{4j}}{\sum_{j=1}^n \mu(x_2^{4j})} - 0,03 \frac{\sum_{j=1}^n \mu(x_3^{4j})x_3^{4j}}{\sum_{j=1}^n \mu(x_3^{4j})} - 0,134 \frac{\sum_{j=1}^n \mu(x_4^{4j})x_4^{4j}}{\sum_{j=1}^n \mu(x_4^{4j})}.$$

If $x_1^5=H$ and $x_2^5=C$ and $x_3^5=B$ and $x_4^5=C$

$$\text{then } y_5^f = 0,202 - 0,10 \frac{\sum_{j=1}^n \mu(x_1^{5j})x_1^{5j}}{\sum_{j=1}^n \mu(x_1^{5j})} - 0,08 \frac{\sum_{j=1}^n \mu(x_2^{5j})x_2^{5j}}{\sum_{j=1}^n \mu(x_2^{5j})} - 0,04 \frac{\sum_{j=1}^n \mu(x_3^{5j})x_3^{5j}}{\sum_{j=1}^n \mu(x_3^{5j})} - 0,12 \frac{\sum_{j=1}^n \mu(x_4^{5j})x_4^{5j}}{\sum_{j=1}^n \mu(x_4^{5j})}.$$

- The risk assessment model, the output of which is expressed by nonlinear dependence.

If $x_1^1=H$ and $x_2^1=H$ and $x_3^1=B$ and $x_4^1=C$

then

$$y_1^f = 0,33 - 0,05 \frac{\sum_{j=1}^n \mu(x_1^{1j})x_1^{1j}}{\sum_{j=1}^n \mu(x_1^{1j})} - 0,02 \frac{\sum_{j=1}^n \mu(x_2^{1j})x_2^{1j}}{\sum_{j=1}^n \mu(x_2^{1j})} - 0,21 \frac{\sum_{j=1}^n \mu(x_3^{1j})x_3^{1j}}{\sum_{j=1}^n \mu(x_3^{1j})} - 0,1 \frac{\sum_{j=1}^n \mu(x_4^{1j})x_4^{1j}}{\sum_{j=1}^n \mu(x_4^{1j})} +$$

$$+ 0,003 \left[\frac{\sum_{j=1}^n \mu(x_1^{1j}) x_1^{1j}}{\sum_{j=1}^n \mu(x_1^{1j})} \right]^2 - 0,004 \left[\frac{\sum_{j=1}^n \mu(x_2^{1j}) x_2^{1j}}{\sum_{j=1}^n \mu(x_2^{1j})} \right]^2 + 0,007 \left[\frac{\sum_{j=1}^n \mu(x_3^{1j}) x_3^{1j}}{\sum_{j=1}^n \mu(x_3^{1j})} \right]^2 + 0,001 \left[\frac{\sum_{j=1}^n \mu(x_4^{1j}) x_4^{1j}}{\sum_{j=1}^n \mu(x_4^{1j})} \right]^2$$

If $x_1^{16}=C$ and $x_2^{16}=B$ and $x_3^{16}=H$ and $x_4^{16}=C$

then

$$y_{16}^f = 0,184 - 0,007 \frac{\sum_{j=1}^n \mu(x_1^{16j}) x_1^{16j}}{\sum_{j=1}^n \mu(x_1^{16j})} - 0,005 \frac{\sum_{j=1}^n \mu(x_2^{16j}) x_2^{16j}}{\sum_{j=1}^n \mu(x_2^{16j})} - 0,003 \frac{\sum_{j=1}^n \mu(x_3^{16j}) x_3^{16j}}{\sum_{j=1}^n \mu(x_3^{16j})} - 0,09 \frac{\sum_{j=1}^n \mu(x_4^{16j}) x_4^{16j}}{\sum_{j=1}^n \mu(x_4^{16j})} +$$

$$+ 0,002 \left[\frac{\sum_{j=1}^n \mu(x_1^{16j}) x_1^{16j}}{\sum_{j=1}^n \mu(x_1^{16j})} \right]^2 - 0,0009 \left[\frac{\sum_{j=1}^n \mu(x_2^{16j}) x_2^{16j}}{\sum_{j=1}^n \mu(x_2^{16j})} \right]^2 + 0,0005 \left[\frac{\sum_{j=1}^n \mu(x_3^{16j}) x_3^{16j}}{\sum_{j=1}^n \mu(x_3^{16j})} \right]^2 + 0,0015 \left[\frac{\sum_{j=1}^n \mu(x_4^{16j}) x_4^{16j}}{\sum_{j=1}^n \mu(x_4^{16j})} \right]^2$$

If $x_1^{27}=B$ and $x_2^{27}=B$ and $x_3^{27}=C$ and $x_4^{27}=C$

then

$$y_{27}^f = 0,17 - 0,003 \frac{\sum_{j=1}^n \mu(x_1^{27j}) x_1^{27j}}{\sum_{j=1}^n \mu(x_1^{27j})} - 0,001 \frac{\sum_{j=1}^n \mu(x_2^{27j}) x_2^{27j}}{\sum_{j=1}^n \mu(x_2^{27j})} - 0,07 \frac{\sum_{j=1}^n \mu(x_3^{27j}) x_3^{27j}}{\sum_{j=1}^n \mu(x_3^{27j})} - 0,09 \frac{\sum_{j=1}^n \mu(x_4^{27j}) x_4^{27j}}{\sum_{j=1}^n \mu(x_4^{27j})} +$$

$$+ 0,01 \left[\frac{\sum_{j=1}^n \mu(x_1^{27j}) x_1^{27j}}{\sum_{j=1}^n \mu(x_1^{27j})} \right]^2 - 0,0005 \left[\frac{\sum_{j=1}^n \mu(x_2^{27j}) x_2^{27j}}{\sum_{j=1}^n \mu(x_2^{27j})} \right]^2 + 0,0002 \left[\frac{\sum_{j=1}^n \mu(x_3^{27j}) x_3^{27j}}{\sum_{j=1}^n \mu(x_3^{27j})} \right]^2 + 0,0024 \left[\frac{\sum_{j=1}^n \mu(x_4^{27j}) x_4^{27j}}{\sum_{j=1}^n \mu(x_4^{27j})} \right]^2.$$

- The risk assessment model, the output of which is expressed by fuzzy term.

If $x_1^1=H$ and $x_2^1=H$ and $x_3^1=H$ and $x_4^1=H$ with the weight 0.5

and $x_1^1=C$ and $x_2^1=H$ and $x_3^1=H$ and $x_4^1=H$ with the weight 0.5

Then $y_1^f=B$.

If $x_1^2=H$ and $x_2^2=H$ and $x_3^2=H$ and $x_4^2=C$ with the weight 0.33

or $x_1^2=H$ and $x_2^2=H$ and $x_3^2=H$ and $x_4^2=B$ with the weight 0.33

or $x_1^2=H$ and $x_2^2=H$ and $x_3^2=C$ and $x_4^2=H$ with the weight 0.33

Then $y_2^f = BC$.

If $x_1^3 = H$ and $x_2^3 = H$ and $x_3^3 = H$ and $x_4^3 = B$ with the weight 0.33

or $x_1^3 = H$ and $x_2^3 = H$ and $x_3^3 = C$ and $x_4^3 = C$ with the weight 0.33

or $x_1^3 = H$ and $x_2^3 = H$ and $x_3^3 = C$ and $x_4^3 = B$ with the weight 0.33

Then $y_3^f = C$.

If $x_1^4 = H$ and $x_2^4 = B$ and $x_3^4 = C$ and $x_4^4 = C$ with the weight 0.5

or $x_1^4 = H$ and $x_2^4 = B$ and $x_3^4 = C$ and $x_4^4 = B$ with the weight 0.5

Then $y_4^f = HC$.

If $x_1^5 = C$ and $x_2^5 = B$ and $x_3^5 = C$ and $x_4^5 = B$ with the weight 0.33

or $x_1^5 = B$ and $x_2^5 = B$ and $x_3^5 = C$ and $x_4^5 = B$ with the weight 0.33

or $x_1^5 = B$ and $x_2^5 = B$ and $x_3^5 = B$ and $x_4^5 = B$ with the weight 0.33

Then $y_5^f = H$.

During construction models used the functions of the facilities of the following type [3]:

$$\mu^k(x_i^j) = \frac{1}{1 + \left(\frac{x_i^j - c_k^j}{\sigma_k^j} \right)^2}.$$

The results of the research produced a forecast evaluation of the risk of shortfalls in crop, based on the creation of approximating models with the use of training and testing data on the risk (Figure 1)

Here FACT - the actual value of the risk;

Model1 – risk, defined by fuzzy model 1;

Model2 – risk, defined by fuzzy model 2;

Model3 – risk, defined by fuzzy model 3.

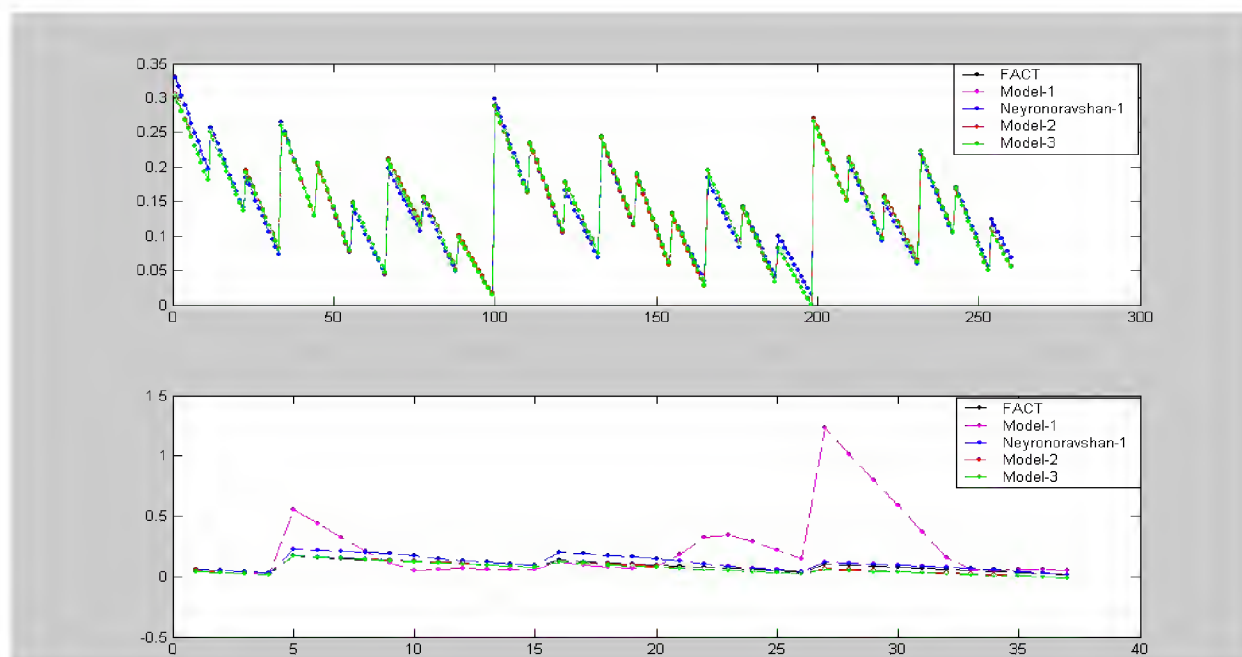


Figure 1: Schedule of Risk Assessment for Training and Testing Data

CONCLUSIONS

The results showed high efficiency of the proposed algorithm for solving decision-making on forecasting, classification and measurement of poorly formalized processes described by fuzzy models [5].

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